A Model of Random Geographical Routing — An Assignment in the Nicta Short Course in Network Analysis

Olivier Mehani
National ICT Australia, and
University of New South Wales, Australia, and
Mines ParisTech, CAOR — Centre de Robotique, Mathématiques et Systèmes, France,
olivier.mehani@nicta.com.au

***Abstract*—We have modelled a random geographical routing model using a discrete time Markov chain. We statistically derive the TTL to be assigned to sent packets depending on such parameters as the lattice size, routing bias or existence of diagonal routes. We also compare these results to previous simulation experiments and find that they closely match.**

***Index Terms*—geographical routing, random routing, simulation, analytical model, short course, assignment**

I. Introduction

A simple yet interesting model for random geographical routing has been introduced in [1]. Simulations based on this model have shown interesting properties linking lattice size, TTL of packets and routing bias [2]. Interestingly, it has also been shown that the possibility of forwarding along diagonal routes does not significantly improve performance.

In this paper, we propose a discrete time Markov chain to model the same system, as suggested in [3]. We use it to verify, and confirm, the results previously found in simulation.

In the following, we first quickly present how the Markov model of a network with random geographical routing has been derived, and how it is used. We then present how various parameters of the system (bias, lattice size, diagonal routes) influences the time-to-live (TTL) new packets need to have to allow a 95% delivery rate.

II. Discrete Time Markov Model

We consider a lattice of $S \times S$ nodes. Each of them can only communicate with its direct neighbors, including the diagonal ones in identified cases. We are interested in the way a packet is forwarded from its source to its destination. For simplicity, we only consider the case where the destination is the furthest away from the source. Specifically, the source is assumed to be node $(0, 0)$, while the destination is node (S, S).

The state of the system is represented by the coordinates of the node which is currently forwarding the packet. Therefore, the initial state of the system is $(0, 0)$. Transition matrices have been derived from biases attributed

$$\begin{vmatrix} b+1 & 2b & b+1 \\ 2 & & 2 \\ 2 & 2 & 2 \end{vmatrix} \qquad \begin{vmatrix} 2 & b+1 & 2b \\ 2 & & b+1 \\ 2 & 2 & 2 \end{vmatrix}$$

(a) (b)

Fig. 1. The weights attributed to neighbors for routing selection with bias b for two cases: (a) the destination is towards the North or, (b) in the North-West direction [2].

as per Fig. 1. For each node, weights for impossible transitions (*e.g.* forwarding to the left when already in one of the leftmost nodes) have first been removed. The sum of the remaining weigths has then been used to derived transition probabilities from the each link. (1) is an example transition matrix for a 2×2 lattice with routing bias 2.

$$P = \begin{bmatrix} 0 & 0.5 & 0.5 & 0 \\ 0.3333333 & 0 & 0 & 0.6666667 \\ 0.3333333 & 0 & 0 & 0.6666667 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad (1)$$

The model has been implemented using Scilab [4]. It uses an simple iterative algorithm to compute the next state from the previous state.

Input: Initial state vector I of size S
Output: Transition matrix P of size $S \times S$
Output: Desired delivery probability p_d
$St \longleftarrow I$;
while $St_{S,S} < p_d$ **do**
| $St \longleftarrow St^T \cdot P$;
end

Algorithm 1: The transitions of our Markov model are iterated until the desired delivery probability is observed in the sink state.

In the next section, we present analytical results derived from this model.

III. Influence of System Parameters on the TTL

We consider a default system with a 4×4 lattice, a routing bias $b = 2$ and no diagonal route. We then

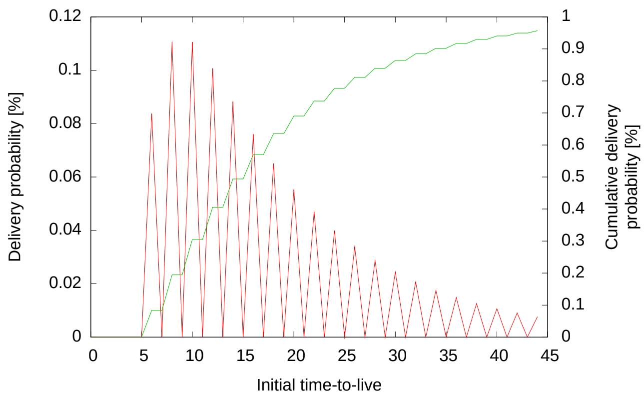

Fig. 2. Delivery probability of a packet given its initial time-to-live.

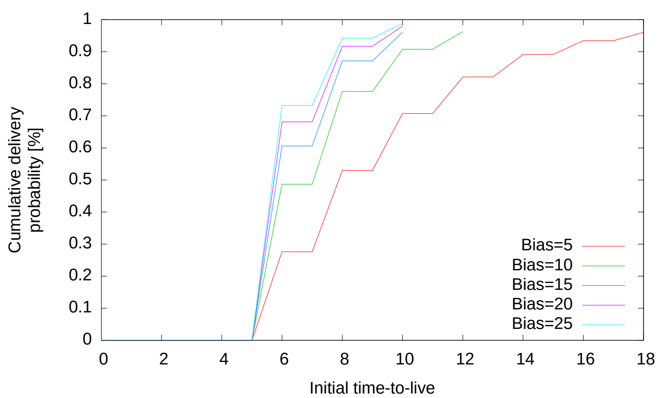

Fig. 3. To be able to use TTLs of the order of 10, it is necessary to have routing biases larger than 20.

vary these parameters and observe the impact of such modification on the observed hop-count for a 95% success probability.

A. *TTL with the Default Parameters*

As shown on Fig. 2, a cumulative success probability of more than 95% is not achieve with TTL lower than 45. This confirms simulated results from [2], thus giving good hints that the presented model is accurate.

B. *Routing Bias*

It has been previously found by simulation that a rather high routing bias (> 20) is needed to allow for reasonable TTL of about 10 hops. This fact is confirmed by our model, as shown on Fig. 3.

C. *Lattice Size*

An interesting property of the random geographical routing system was found to be the seemingly linear relation between the lattice size and the initial time-to-live of a packet. The analytical model confirmed this emerging property, as shown on Fig. 4. It seems, however, that this finding does not hold for small S, as can be observed in the figure by comparing results for sizes $S = 5$ and $S = 10$.

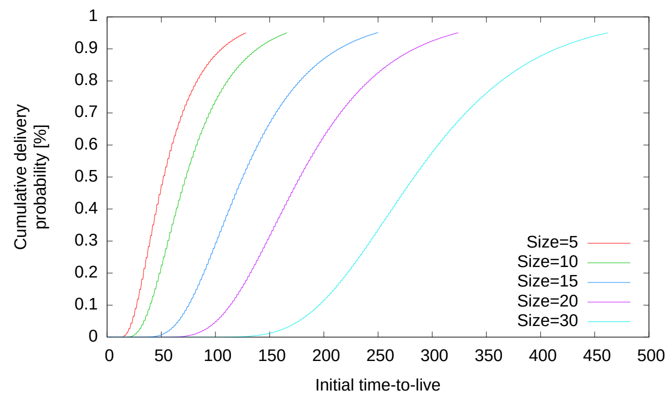

Fig. 4. Comparing TTL for size pairs $(10, 20)$ and $(15, 30)$ shows a linear relation between S and the hop-length.

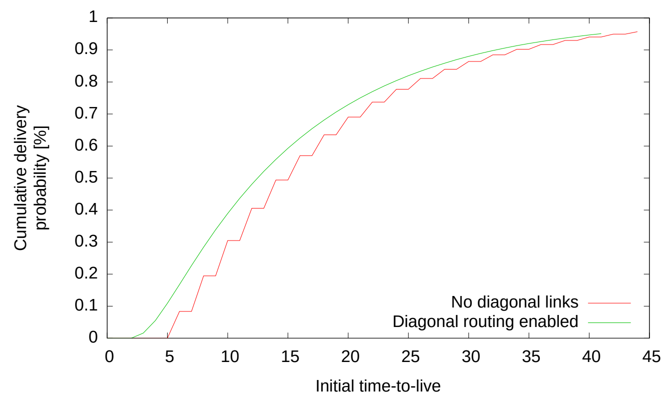

Fig. 5. Enabling forwarding along diagonal links only provides little improvement over the default parameters.

D. *Diagonal Links*

In what we find to be a rather counter intuitive way, allowing diagonal links only added little improvement to the system. Evaluating this modification using the Markov model confirmed this finding, as is shown on Fig. 5

IV. Conclusion

We introduced a simple discrete time Markov chain to model a routing mechanism based on biased random geographical information. This analytical approach confirmed results which were found in previous simulation-based works.

The mentionned simulations also covered failing nodes, which was not taken into account for the model presented here. Future work should include this possibility, as this allows to model more closely a real network of machines.

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